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MACHINE INVESTMENT AND FACILITIES PLANNING

A THESIS

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MACHINE INVESTMENT AND FACILITIES PLANNING

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SUMMARY

The objective of this study was to develop a model to determine the magnitude of machine-tool investment which minimizes the unit cost for a certain product demand or forecast pattern.

Before a decision can be made to purchase one machine-tool in preference to another, there are fundamental factors, such as physical characteristics, that must be considered. After the preliminary elimination of machine-tools which do not satisfy the required physical characteristics, the next step is to estimate and compare the costs of the remaining alternatives.

The techniques used to select between different alternatives are those of engineering economy and the break-even analysis. However, these techniques did not express the desired quantitative relationships between demand, unit cost and money investment in machine-tools.

Therefore, a model that expresses these desired relationships was derived using the economic balance theory. It was found that certain categories of costs, notably the fixed investment costs, tended to rise while other costs, the variable operating costs, tended to fall with increases in the design variable. The design variable in this case is the machine-tool investment. Since the total cost is the sum of all costs, the problem was one of balancing these opposing movements in cost to achieve a minimum cost design.

In manufacturing operations the following model can be set up: the manufacturing cost = the investment recovery cost + labor and over-

head costs. Solving this relationship results in an equation for the optimal investment as a function of the demand and labor and overhead cost, which is given by:

$$I = \frac{q+1}{}\sqrt{\frac{qLQp}{R}}$$

where Q = demand in units per time period (one year)

L = labor and overhead cost in dollars per minute

I = optimal machine-tool investment in dollars

p and q are constant factors controlling respectively the vertical position and curvature of the production curve.

The second model derived was based upon the production theory. The production function for a process requiring two inputs can be written as $Y = f(X_1, X_2)$ where X_1 = number of manhours, X_2 = capital investment in dollars and Y the output. The cost function for the same process is $C = C_1X_1 + C_2X_2$, where C is the total cost, C_1 the cost of one unit of labor and overhead (in dollars per hour) and C_2 is the capital recovery factor for n years at a minimum attractive rate of return agreed to by management. To obtain the optimal investment, the cost function is minimized subject to the constraint expressed by the production function. The optimal condition was obtained at the point where the cost function was tangent to the production function.

CHAPTER I

INTRODUCTION

Objective

The objective of this study is to develop a model to determine the magnitude of machine-tool investment which minimizes the unit cost for a certain product demand or forecast pattern.

Background

Before a decision can be made to purchase one machine-tool in preference to another, there are some fundamental factors that must be considered. The choice of a machine-tool must take into account the following:

1. The size and shape of the workpiece: Machine-tool specifications generally specify the size of the largest workpiece that can be handled by the machine.

2. Accuracy and surface-quality required: Machine-tools are capable of producing up to a certain accuracy or have process capability ranges. These ranges should be compared with the design tolerance ranges required by the product drawings and process engineering tolerance charting.

3. Strength and power: A heavy machine usually has a heavy motor; not only sufficient power should be provided at the tool but motors should be mounted in such a way as to minimize mechanical vibrations and be easily serviced.

4. Capacity: The individual machine-tool has to be able to match the planned capacity required, that is, provide the required rate of production, or multiple machine-tools must be provided.

5. Speeds and feeds: In selecting a machine, consideration should be given to the number of speed changes and feed ranges available.

6. Lubrication: No machine-tool should be selected that does not incorporate provision for adequate lubrication, where such is required.

7. Safety: Consideration should always be given to the safety of both operator and the machine.

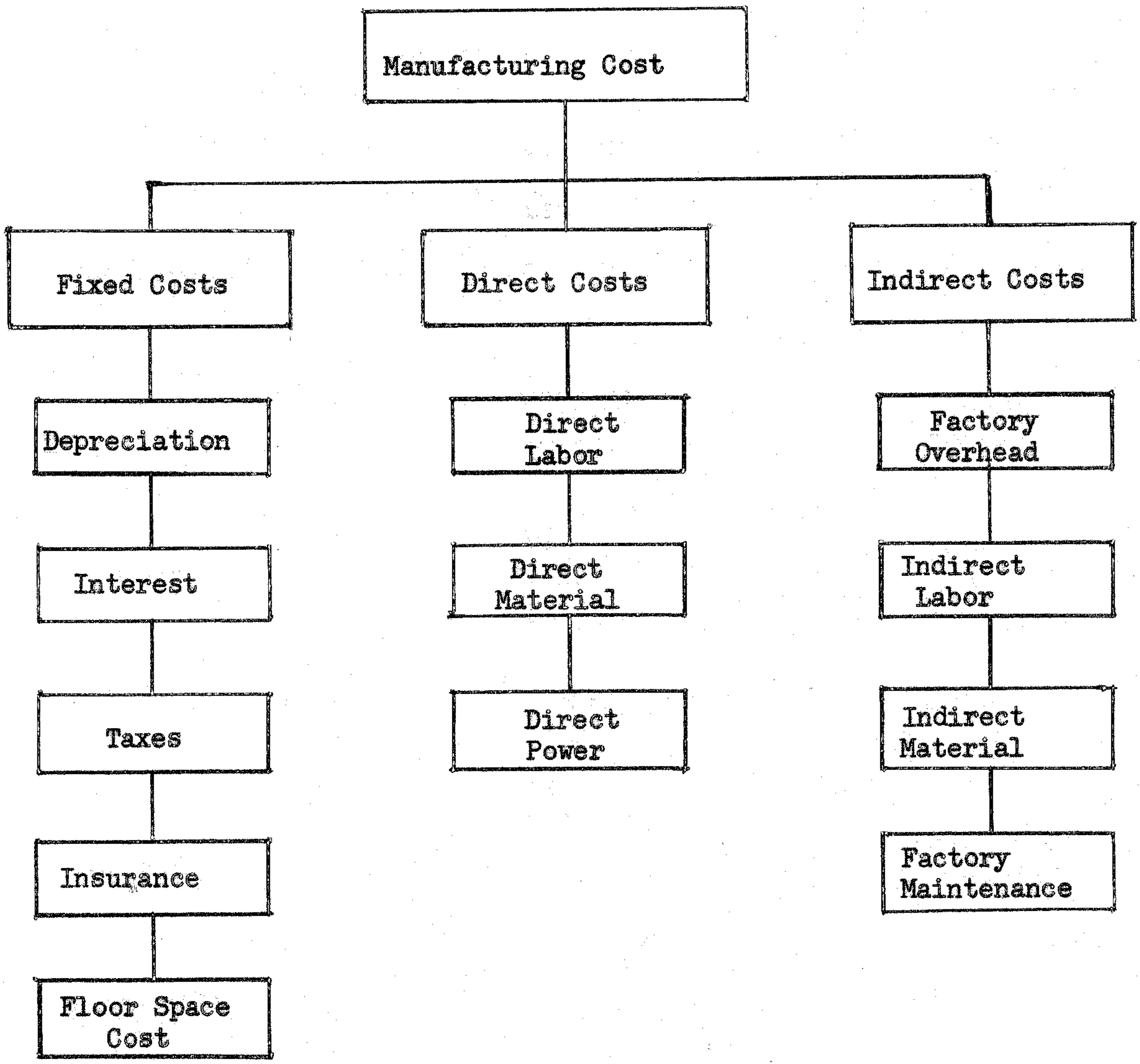
As a result of the general considerations just described, it may be possible to eliminate all but a few types of machine-tools from consideration for a specific machining operation. After the elimination by applying the above principles of meeting specifications, the next step is to estimate and compare costs. In any case the selection of alternatives must be economically sound.

The cost factors, as applied herein, fall into two categories, namely: the investment cost and manufacturing or operating cost. As shown in Table 1, the manufacturing cost can be divided into three groups: fixed, direct and indirect costs. The investment costs are the costs that are incurred as a result of the capital invested in the machine-tool. They are:

1. initial cost of the machine and its accessories,
2. transportation cost,
3. installation cost, and
4. tooling cost.

It should be noted that the machine-tool and its tooling are investments

Table 1. The Different Cost Factors Making up Manufacturing Cost



usually handled separately because of their different prospective service lives. The tooling is amortized over a shorter period of time, whereas the machine-tool is depreciated over a longer time period. Time element, then, is the major difference between the two types of investment.

Survey of Literature

The normal methods in use to select between different machine-tools are the engineering economy techniques employed when money has a time value (e.g., annual cost, present worth, capitalized cost and rate of return) and the break-even analysis techniques where the time value of money is not a factor.

One of the comparative cost analyses as described by Grant (1) consists in calculating the annual cost for the alternatives, taking into account all the manufacturing or operating and ownership costs. It is assumed that the volume of production remains fixed, and only the difference between the annual costs of the alternatives is used as the quantitative criterion for selection. The annual costs compared are the sums of the cost of ownership and the cost of operation for the same time period and interest rate.

The comparison by break-even analysis, as described by Doyle (2) and many others, consists of comparing alternatives by a break-even graph or a schematic model. The manufacturing cost is drawn on the graph as a function of the production in units. The break-even point or the equal cost point for an operation is the quantity at which the total and unit costs are the same for the machine-tools compared. One of the machine-tools is the more economical for smaller production quantities and the other for larger production quantities. A typical graph is shown in Fig. 1.

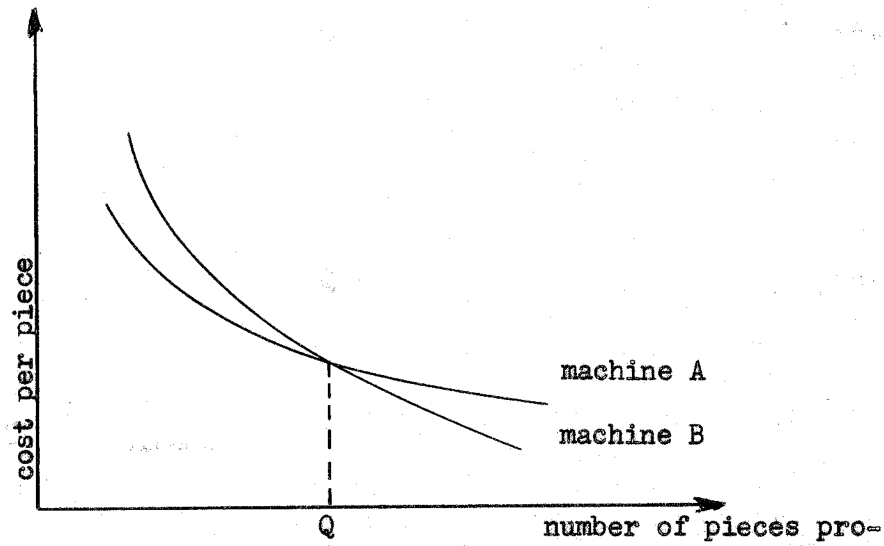


Figure 1. A Typical Equal Cost Situation

The first quantitive relationship between demand, unit cost and investment in tooling plans was presented by Wu (3) in his article, "Optimal Tooling Decision". In this paper he developed a model for:

> (1) the tool investment and hence the tooling plan to be used on a machine, which minimizes the unit cost at a fixed and known demand, and (2) the tool investment which would minimize the losses under conditions where demand was uncertain.

Following Wu's procedure, the economic balance approach for machine-tool investment was developed as indicated herein in Chapter II.

In his book "Investment and Production", Smith (4) describes a method for determining the optimal investment, using the production function and cost function. This is a method that is often used in the chemical industry, as noted by Schweyer (5). Following this theory, the production theory has been used herein in Chapter III as another approach to the machine-tool selection and investment problem.

Additional books were consulted about the topic, which are listed in the bibliography.

CHAPTER II

THE ECONOMIC BALANCE APPROACH

It is known that certain categories of costs, notably the "fixed" investment costs, tend to rise, while other costs, the "variable" operating costs, tend to fall with the design variable. Since the total cost is the sum of all costs, the problem is to balance these opposing changes in cost to achieve a minimum cost design. The economic balance or minimum point occurs where the rate of rise in the investment costs is equal to, or just balanced by, the rate of decline in the variable cost.

The manufacturing cost is the sum of different costs, namely, the fixed, direct and indirect costs as shown in Table 1. However, one can easily make the assumption that most of these costs do not vary from one production plan to another; for example, the cost for direct material, building rental and lubrication will be the same (or approximately) whether the piece is turned on a turret lathe or on an engine lathe.

The costs that vary considerably from one production method to another are the investment cost and the direct labor cost. As a result, one can abstract the following model: The manufacturing cost = the investment recovery cost + the labor and overhead cost.

$$C = Lt + \frac{IR}{Q} \tag{1}$$

C = the manufacturing cost per unit of production; this is only the variable component of the manufacturing cost, because the other costs are assumed to stay the same in the different tooling plans.

L = labor and overhead rate in \$ per minute

t = the production or process time per piece (in min.)

I = the investment in equipment or machine tools expressed in dollars

R = capital recovery factor

Q = number of items to be produced per year

In applying this model four assumptions shall be made:

1. that other cost factors not included in the model are the same or constant in each plan,
2. that the machines are depreciated over the same period or all machines have equal estimated life periods,
3. that the interest rates are the same throughout, and
4. that the salvage values of the machines are zero.

Product-Investment Relationship

When a part is hand-made, the tooling is hand-tooling and its cost low while the process time taken to make the part is very high. If we invest in simple machinery, such as an engine-lathe, then we know that the process time will decrease over that taken by hand-making. Accordingly production times are further lowered with higher machine tool investments (such as a multiple-spindle automatic machine). Therefore, the production or process time has an inverse relationship to the dollar investment in machine tools. A study of several machining plans for the production of a part has shown that in each case the production time--machine investment relationship can be approximated by a curve (fitting a curve to the empirical data) of the general shape shown in Figure 2.

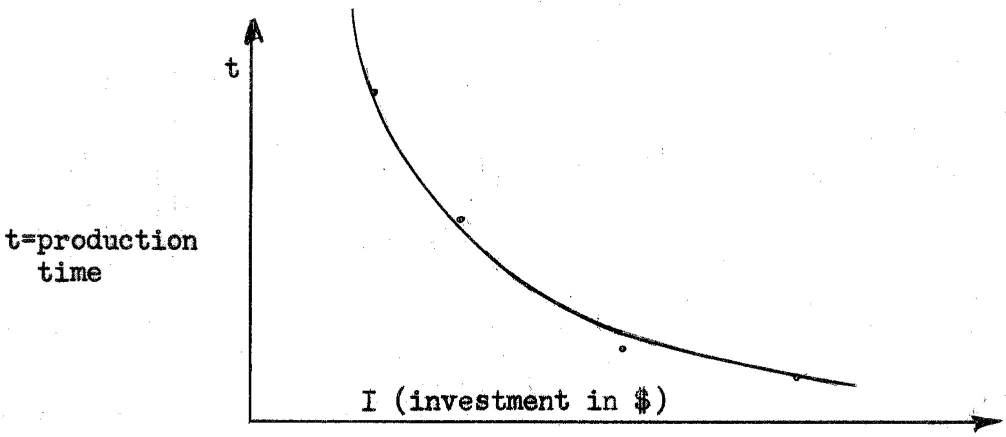


Figure 2. Approximation of the Relationship Between Machine-Tool Investment and Production Time

The general equation for curves of this form can be written as

$$t = \frac{p}{I^q} \tag{2}$$

The factor p in this formula controls the vertical position of the curve and q the degree of curvature. While t is a continuous function of I, I itself is not continuous in actual experience. No known machining plan exists corresponding to many values of I. Of course, formula (2) can only be employed within certain limitations. On one side, the curve is asymptotic to the t axis, while on the other side, the curve will never be asymptotic to the I axis; the larger the investment, the shorter the process time becomes, but there is a physical limit which makes the time t tend to zero but never equals it. Nevertheless the range in which the formula will be used will not be influenced by this limitation.

Substituting (2) in (1) gives

$$C = \frac{Lp}{I^q} + \frac{IR}{Q} \tag{3}$$

Labor and Overhead Cost

The first term in equation (3) covers the labor and overhead cost, in terms of I, p and q, per unit of production. The factor L (labor and overhead rate) can be written as follows

$$L = \frac{r(1+k)}{60}$$

r = labor rate per hour in $

divided by 60 to obtain the rate per minute

k = overhead factor in percentage of direct labor

(overhead here has the same significance as used in cost accounting, where the distribution of overhead is set as a percentage of direct labor, but excludes any machine depreciation)

Investment Recovery Cost

The second term in equation (3) is the investment recovery cost $\frac{IR}{Q}$ per unit of output.

I = amount invested in $ in the machine tools

Q = the quantity produced per year

R = capital recovery factor. f.e. for n=12 year and i=20% (i=interest) found in the appropriate compound interest table (see Grant (1)) gives

R = 0.22526

Any kind of investment must be justified by the prospect of a return. The return on an investment is the capital recovery on the money invested in the asset. The capital recovery cost includes recovery of the principal invested in the machine itself as well as an expected return on the money invested in the machine. The investment in the machine is recovered over a specified number of years, taken here as a period of twelve years, as at present set for machine-tools of these types by the Internal Revenue Service.

The capital recovery factor can be written as:

$$(CRF)^{n}_{i\%} = R = \frac{i(1+i)^{n}}{(1+i)^{n}-1}$$

where n = number of years of useful life

i = rate of return (set by management as being the minimum attractive return (or interest rate) established by policy for investments of this type)

Economic Balance Point

Considering equation (3): $C = \frac{Lp}{I^{q}} + \frac{IR}{Q}$, it should be noted that the labor and overhead costs decrease non-linearly while the capital recovery cost increases linearly with investment (I) as illustrated in Fig. 2.

The economic balance or minimum point occurs where the rate of rise in the capital recovery cost per unit is equal to the rate of decline in the labor and overhead cost per unit.

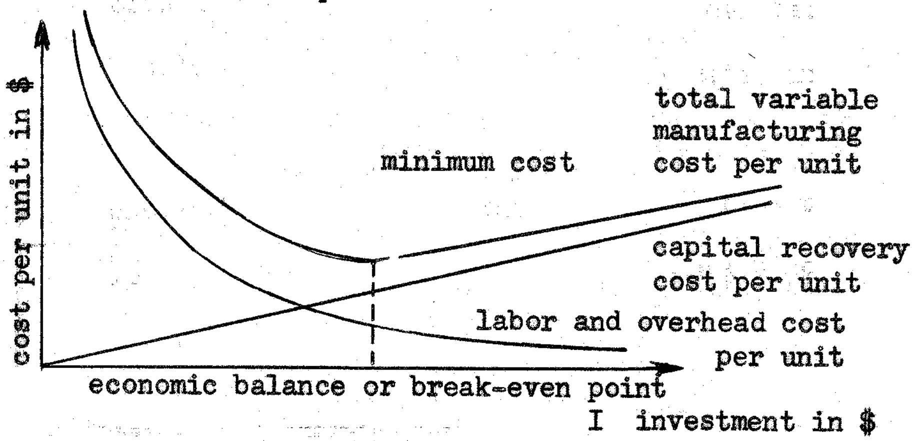


Figure 3. The Unit Cost as a Function of the Investment

This minimum point can be obtained by equating to zero the first derivative of C (total variable manufacturing cost) with respect to I (investment in dollars).

$$C = \frac{Lp}{I^q} + \frac{IR}{Q}$$

$$\frac{dC}{dI} = -\frac{qLp}{I^{q+1}} + \frac{R}{Q} = 0$$

$$I = \sqrt[q+1]{\frac{qLpQ}{R}} \qquad (4)$$

I is the optimum investment in dollars which minimizes the manufacturing cost at a production level Q. This solution holds when $t = \frac{p}{I^q}$ holds. It was stated previously that in actual experience I is not continuous. As a result the solution gives only an ideal or approximate answer. The final decision for I would be an approximation of the calculated optimal amount, but it nevertheless will constitute a decision rule.

Example

It is anticipated that the yearly demand for the part shown in Fig. 4 will be 8,000 pieces. The manufacturing department has a labor rate of \$1.50 an hour, and a general overhead factor of 150 per cent, making a total labor and overhead charge of $r(1+1.5)=\$1.5(1+1.5)=\$3.75$ per hour. The minimum attractive rate of return desired is 20 per cent before taxes. The question is, how much should be invested in a machine to manufacture this part?

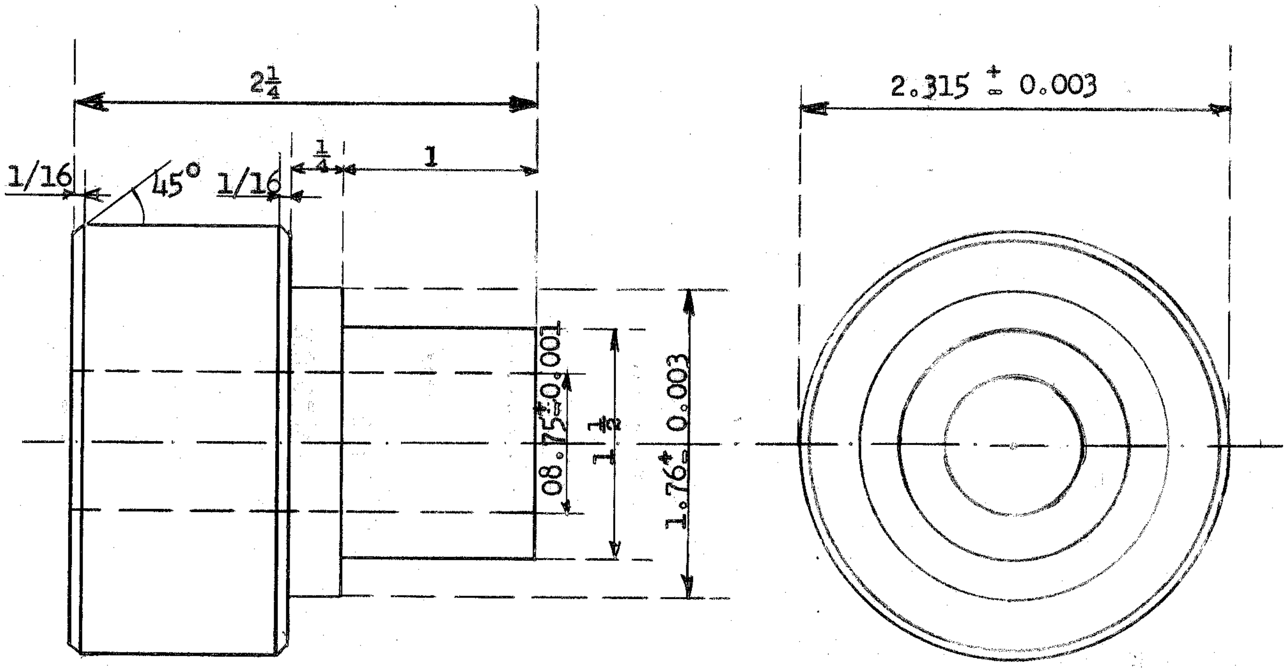


Figure 4. A Starter Pinion Gear Blank

A tabular analysis, showing four different production plans, is shown in Table 2. We find first the cost of the machine plus the cost of the necessary accessories. Next we find the operation time--this means the time it takes the machine to manufacture the part. The labor and overhead cost is found by multiplying the labor and overhead rate by the operation time, i.e., using an engine lathe for example, we have

$$\frac{15\text{x}\$3.75}{60} = \$0.9375 \text{ labor and overhead cost per unit}$$

The annual capital recovery cost is obtained by multiplying the investment in dollars with the capital recovery factor for n=12 years and i=20 per cent, e.g., 4,000.00 x 0.22526 = \$901.04. To obtain the capital recovery cost per piece we divide by the number of items to be produced per year Q=8,000. Adding the labor and overhead cost per unit to the capital recovery cost per unit gives the total variable cost per unit.

Table 2. Comparison of Production Plans

	Engine Lathe	Turret Lathe	Single-spindle Automatic	Multi-spindle Automatic
Cost of machine	$3200.00	$6500.00	$10,750.00	$17,325.00
Cost of Accessories	800.00	1234.75	750.00	750.00
Total Cost	$4000.00	$7734.75	$11,500.00	$18,075.00
Operation time per piece in minutes	15	6.5	3.0	1.2
Labor and Overhead cost per piece (1)	$0.9375	$0.4063	$0.15 *	$0.06 *
Annual Capital Recovery cost R=0.22526	$901.04	1,742.33	2,590.49	$4,071.57
Capital Recovery Cost per piece/year (2)	$0.1126	.2178	0.3238	0.5089
Total Variable Cost per piece (1) + (2)	$1.05	0.624	0.4738	0.5689

* The labor rate is $1.50 per hour on turret and engine lathes but is $0.75 per hour on automatics where one man attends two machines. The overhead however stays equal $2.50 per hour, thus labor and overhead cost for automatics is $3.00 per hour.

Basic data in this table were obtained from E. L. Murray, "Machine Tool Selection in the Turning Field", The Tool Engineer, June, 1949 (7).

The values obtained in Table 2 can be plotted on a graph as shown in Figure 5. The investment is plotted on the abscissa and the unit cost per piece on the ordinate. Drawing a curve and/or a line through the calculated points gives respectively the capital recovery line, the labor and overhead cost curve, and the total variable cost per unit.

The machines are assumed to work at the cutting speed that gives minimum cost and not that for maximum production. There is a considerable difference in output when using cutting speed for maximum production instead of the cutting speed for minimum cost. When using the cutting speed for maximum production, the cost of manufacturing will be much higher because the tools wear out faster and will have to be replaced more often. (However, it is feasible in some cases, like war conditions, to use the cutting speeds that give maximum production).

Before one can evaluate I using model (4) given by equation

$$I = \sqrt[q+1]{\frac{qLpQ}{R}}$$

it is necessary to determine the values for p and q. Model (2) $t = \frac{p}{I^q}$ can be restated as follows, taking the logrithm of both sides of the equation, $q \log I + \log t = \log p$. In order to obtain the equation of a line on logarithmic paper let:

$$\log I = x$$

$$\log t = y$$

$$\log p = a$$

and it follows

$$q\,x + y = a$$

$$\text{or } y = a - q\,x \qquad (2a)$$

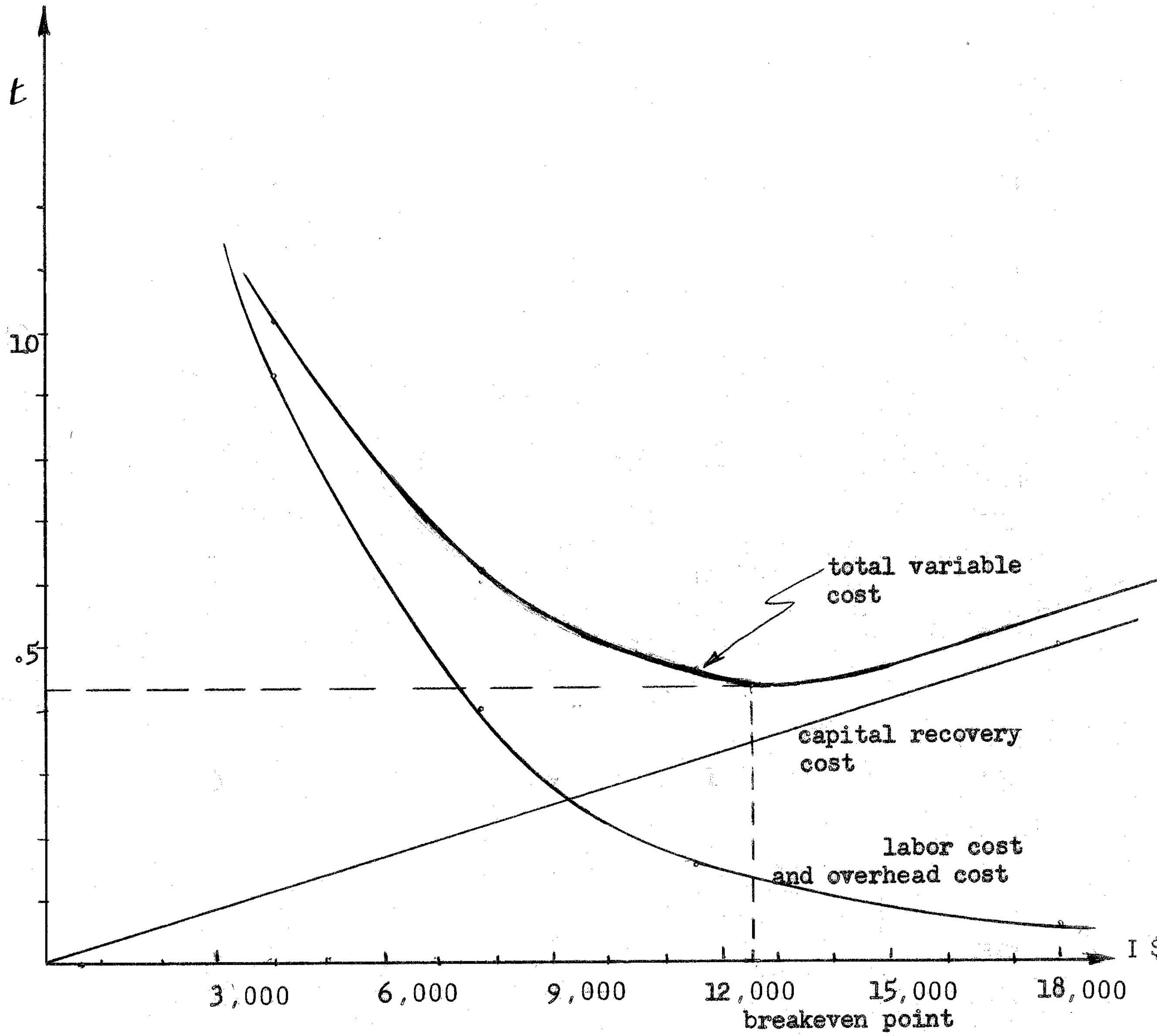


Figure 5. Breakeven Point for Starter Pinion Gear

The problem then is one of statistically fitting a line through the points. The value of a and q can be determined by means of the following equations:

$$\text{(a)}\quad \sum y = b\sum x + an$$

$$\text{(b)}\quad \sum xy = b\sum x^2 + a\sum x$$

$$\text{(c)}\quad y = a + bx \quad \text{where } b = -q$$

t(min	I $	log t=y	log I=x	xy	x^2
15	4,000	1.17609	3.60206	4.236347	12.974836
6.5	7,735	0.81291	3.88846	3.160980	15.120121
3.0	11,500	0.47712	4.06070	1.937441	16.489284
1.2	18,075	0.07918	4.25708	0.337075	18.122730
		2.54530	15.80839	9.67184	62.70697

substituting these values in (a) and (b) gives

$$2.54530 = 15.80830b + 4a$$

$$9.67184 = 62.70697b + 15.80830a$$

solving this equation we obtain

$$b = -1.67$$

$$a = 7.23629$$

hence equation (c): $y = 7.23629 - 1.67x$, which is shown in Figure 6. Comparing this equation with equation (2a) gives us

$$q = 1.67$$

$$a = \log p = 7.23629$$

$$\text{or } p = 1{,}723 \cdot 10^4$$

Substituting the values of p and q in equation (4),

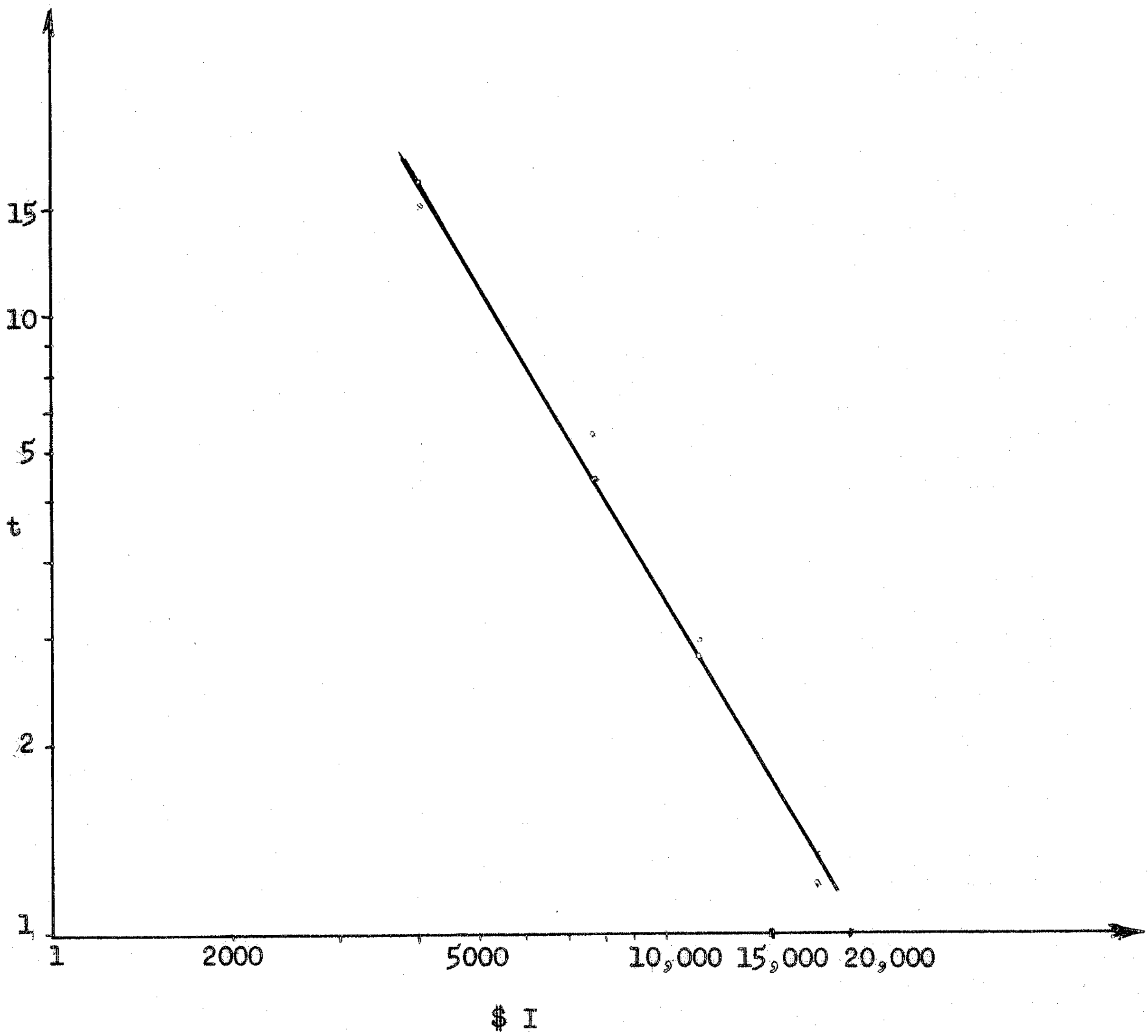


Figure 6. Fitting a Line Through Points

$$I = \underline{2.67}\sqrt{\frac{1,723 \cdot 10^4 \cdot 1.67 \times 3.75 \times 8,000}{60 \times 0.22526}}$$

$$I = \underline{2.67}\sqrt{72,043.83 \times 10^6}$$

$$\log I = \frac{1}{2.67}\left[\log 72,043.83 + 6\right]$$

$$= 4.06651$$

$$I = 11,654 \$$$

This value is of course a theoretical investment cost. The actual or final investment cost will be an approximation of this investment, here 11,500 $ and the related unit cost per piece would be $0.4738 (Values from Table 2).

CHAPTER III

THE PRODUCTION THEORY APPROACH

The Production Curve

Consider a production process requiring two inputs. X_1 and X_2 represent the respective quantities of the two inputs and Y is the quantity of output. The production function can be written as $Y = f(X_1, X_2)$. This function provides a quantitative description of the various quantities of the two inputs which can be employed to produce output Y.

A distinction has to be made between two decisions, namely the engineering and the economic decision. The former can be made on purely technical grounds without any knowledge of costs whatsoever. If, for the same output, one of the inputs can be decreased without a corresponding increase in any other input, then a decision in favor of the modification can be made on purely engineering grounds without the necessity of knowing the input prices. Hence any action which reduces one input without changing any other requirement of a process will lower cost regardless of the price of that input.

The production function $Y=f(X_1,X_2)$ presupposes that all such engineering decisions have been made, so that the best assumed engineering technology remains. But there are still a number of input combinations possible, which have the characteristic that output cannot be maintained at a given or predetermined level when one input is reduced unless there is an increase in some other inputs. The choice among the remaining input combinations is an economic decision, in the sense that the decision requires knowledge of input prices.

Characteristics of the Production Function

The production function as defined above will, in general, exhibit the following characteristics:

1. If one input is held constant while another is increased (decreased), the output will increase (decrease).
2. If the output is held constant, a decrease (increase) in one input will require an increase (decrease) in the other input.
3. If Y (output) is held constant the rate at which X_1 (input 1) replaces X_2 (input 2), increases as X_1 increases, mathematically $\left[\frac{\partial}{\partial X_1}\left(\frac{\partial X_2}{\partial X_1}\right)>0\right]$, or one can say that the production function is convex to the origin in the plane X_1 and X_2.

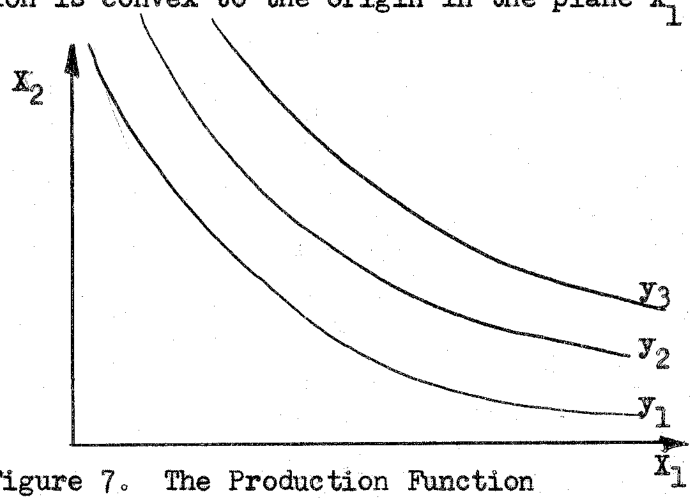


Figure 7. The Production Function

These characteristics can be summarized with an iso-product chart. An iso-product chart is a family of curves on which each curve corresponds to a different constant level of output. Such a curve, therefore, connects all those combinations of X_1 and X_2 that are required to make a specified quantity of output. Figure 7 shows an iso-product chart for output levels Y_1, Y_2, Y_3.

The Cost Function and Iso-Cost Contours

If you have a process with two inputs and C_1 is the cost of input number 1, and C_2 the cost of input number 2, then the total cost function can be written as:

$$C = C_1X_1 + C_2X_2 \qquad (5)$$

The product C_1X_1 is the total expenditure on X_1 units of input number 1, while the product C_2X_2 is the total expenditure on X_2 units of input number 2.

For the same outlay, Figure 8 represents different combinations of X_1 and X_2 as expressed by equation (6).

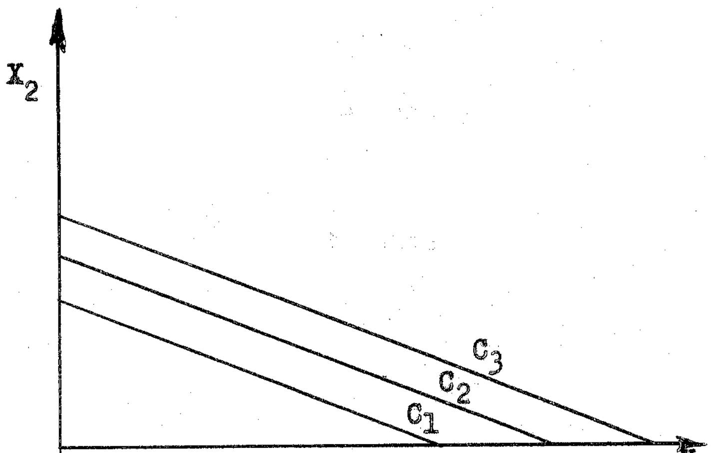


Figure 8. Iso-Cost Contours

It is interesting to note that the intercept of the cost function (equation 6) with the X_1 axis is given by $\frac{C}{C_1}$ and the intercept on the X_2 axis by $\frac{C}{C_2}$.

The Minimum Cost Input Relation

Figures 7 and 8 can be combined into Figure 9, to denote both the production function and the cost curve. The production function is given by $Y=f(X_1,X_2)$ and if we let $y'=Y$ represent a required rate of output, then the expression $y'=f(X_1X_2)$ defines one of the family of iso-product contours as shown in Figure 9.

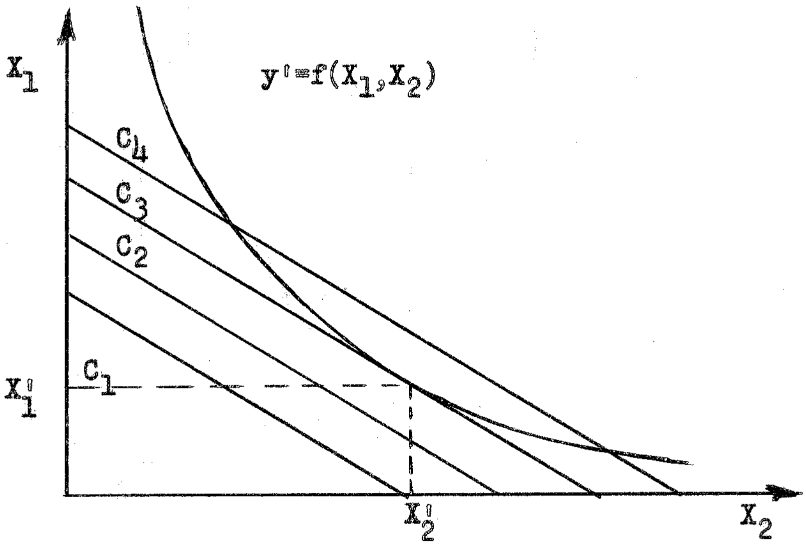


Figure 9. The Minimum Cost Point (X_1', X_2')

This contour represents all the input combinations (X_1, X_2) that will produce y' units of the output.

The next step is to draw some members of the family of the iso-cost lines given by the total cost function:

$$C = C_1X_1 + C_2X_2$$

as shown in Figure 9.

The problem is to choose that input combination (X_1', X_1') which will allow y' units to be produced, and at the same time minimize the total cost C. Figure 9 clearly shows that the objective will be attained where the iso-cost line is tangent to the production curve.

Application to the Machine-tool Investment Problem

The two inputs of the production function $Y=f(X_1,X_2)$ are X_1, the number of manhours, and X_2 the capital investment. In the example $Y=y'$ (output) is 8,000 pieces of the pinion gear shown in Figure 4.

In the cost function, $C=C_1X_1 + C_2X_2$, C_1 represents the labor and overhead cost per hour (\$3.75) and C_2 the capital recovery factor (0.22526). The cost function can thus be written as $C=3.75\ X_1 + 0.22526\ X_2$ (6)

Analytical Solution

First set up the equation of the production function. From Table 2 prepare Table 3 which gives the total time in manhours to produce the output (8,000 pieces) using the four different manufacturing methods.

Table 3. Manufacturing Time

time per piece in minutes	time for 8,000 pieces in hours	investment in dollars
15	2,000	4,000.00
6.5	866	7,734.75
3.0	400	11,500.00
1.2	160	18,075.00

From Table 3, Figure 10 can be drawn.

The problem now is to find an equation $Y=f(X_1,X_2)$, for the curve in Figure 10, plotted from the data of Table 3. Different equations were tested and the following two gave the best fit to the curve:

$$Y = AX_1 + BX_2 + C\sqrt{X_1} + D\sqrt{X_2} + E\sqrt{X_1X_2} \qquad (7)$$

$$Y = K\ X_1\ X_2^q \qquad (8)$$

The advantage in using equation 7 is that it is completely independent of the model built in the second chapter, while equation 8 is based on equation 2 as will be shown later.

Method 1. Equation 7 will be utilized to determine the optimal investment. First determine the value of the different parameters A,B,C, D, and E, utilizing the curve and equation 7. Five linear equations with five unknowns A,B,C,D, and E are obtained thereby, specifically:

$$\begin{aligned}
2{,}000A + 4{,}000B + 2{,}000C + 4{,}000D + 2{,}000 \times 8{,}000E &= 8{,}000\\
1{,}320A + 6{,}000B + 1{,}320C + 6{,}000D + 1{,}320 \times 6{,}000E &= 8{,}000\\
866A + 7{,}735B + 866C + 7{,}735D + 866 \times 7{,}735E &= 8{,}000\\
400A + 11{,}500B + 400C + 11{,}500D + 400 \times 11{,}500E &= 8{,}000\\
160A + 18{,}075B + 160C + 18{,}075D + 160 \times 18{,}075E &= 8{,}000
\end{aligned}$$

To solve these systems of linear algebraic equations a computer program was set up, on the Burroughs 220.*

The solution vector obtained was A = -1.69, B = -0.33, C = 236.05, D = 103.22 and E = -1.55. Substituting these values in equation 7, the equation for the production function was obtained and is given in equation 9.

$$Y = -1.69X_1 - 0.33X_2 + 236.05\sqrt{X_1} + 103.22\sqrt{X_2} - 1.55\sqrt{X_1X_2} \quad (9)$$

In order to find the optimal investment it is necessary to minimize the cost equation (6) subject to the constraint in equation (9) $Y = y'$ (a constant). The mathematical technique used to accomplish this was the Lagrangian multiplier technique. By this method equation (6) is minimized subject to the constraint in equation (9) by minimizing a new expression, say ϕ, of the form:

$$\phi = C_1X_1 + C_2X_2 + \lambda\left[y' - f(X_1,X_2)\right]$$

The notation for the Lagrangian multiplier is λ. The expression of ϕ can be regarded as a function of the three variables X_1, X_2 and λ, and necessary conditions for a minimum of ϕ can be obtained by setting the partial derivatives $\frac{\partial \phi}{\partial X_1}$, $\frac{\partial \phi}{\partial X_2}$, $\frac{\partial \phi}{\partial \lambda}$ equal to zero.

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* Burroughs Technical Bulletin 152 was used to prepare the data charts.

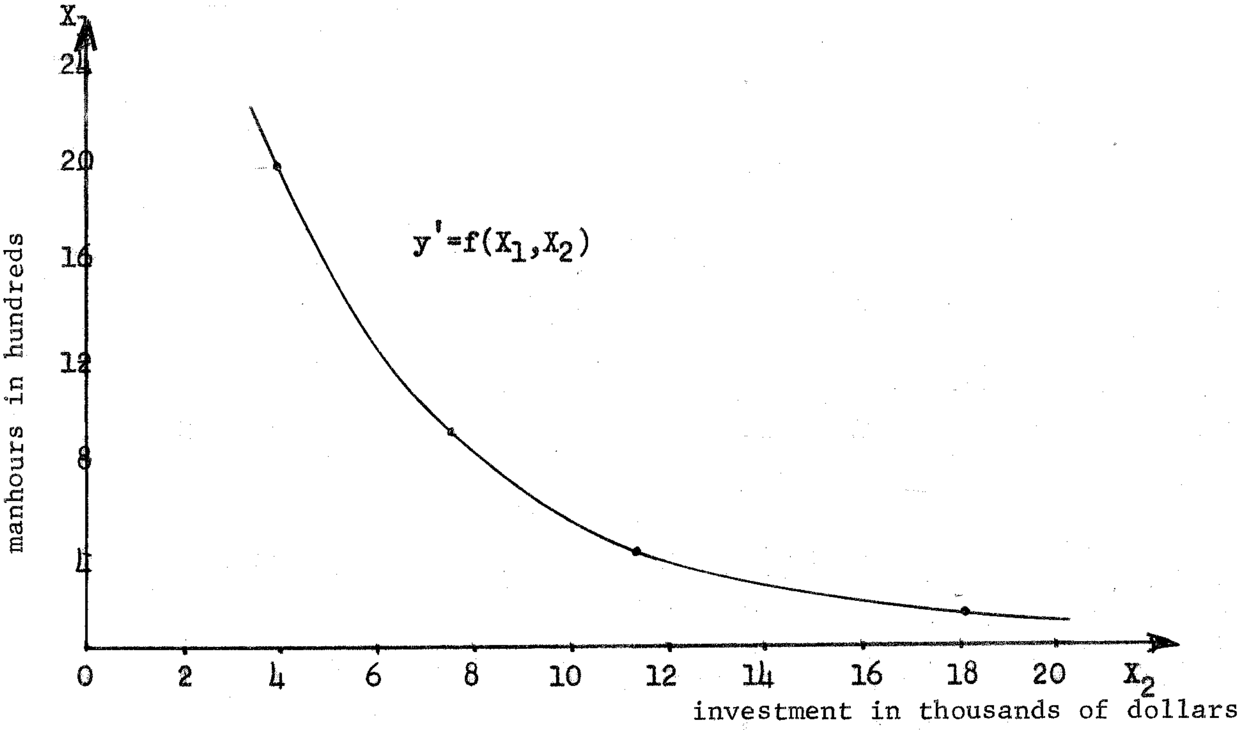

Figure 10. Production Curve for $y = y' = 8{,}000$ Pieces

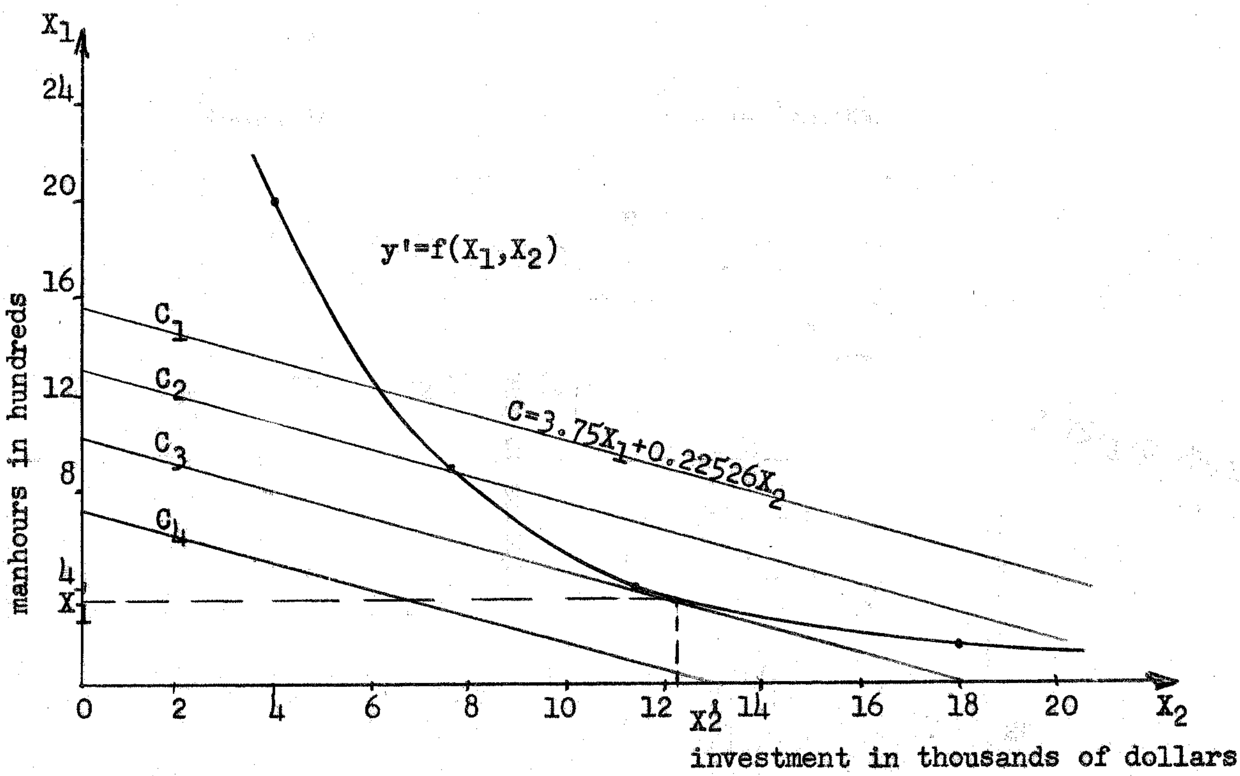

Figure 11. Graphical Solution

$$\frac{\partial \phi}{\partial X_1} = C_1 - \lambda \frac{\partial f}{\partial X_1} = 0$$

$$\frac{\partial \phi}{\partial X_2} = C_2 - \lambda \frac{\partial f}{\partial X_2} = 0 \qquad (10)$$

$$\frac{\partial \phi}{\partial \lambda} = y' - f(X_1, X_2) = 0$$

let $\frac{\partial f}{\partial X_1} = f_1$

and $\frac{\partial f}{\partial X_2} = f_2$

The first two conditions in (10) can be written in the form:

$$\frac{C_1}{f_1} = \frac{C_2}{f_2} = \lambda \qquad (11)$$

The last condition in (10) is simply the production function restraint.

In the form $\frac{C_1}{f_1} = \frac{C_2}{f_2}$ the condition states that cost is at a minimum as input number 1 and input number 2 are employed up to the point at which the marginal worth of input number 1 is equal to the marginal worth of input number 2. This relationship denotes the location of the point of tangency between the iso-product and iso-cost contours. For the example

$$f_1 = \frac{\partial f}{\partial x_1} = -1.69 + 118.025\ X_1^{-\frac{1}{2}} \ -0.775\sqrt{\frac{x_2}{x_1}}$$

$$f_2 = \frac{\partial f}{\partial x_2} = -0.33 + 51.61\ X_2^{-\frac{1}{2}} \ -0.775\sqrt{\frac{x_1}{x_2}}$$

Substituting these values together with the values of $C_1 = 3.75$ and $C_2 = 0.22526$ in equation (11) gives:

$$\frac{3.75}{-1.69+118.025X_1^{-\frac{1}{2}}-0.775\sqrt{\frac{X_2}{X_1}}} = \frac{0.22526}{-0.33+51.61X_2^{-\frac{1}{2}}-0.775\sqrt{\frac{X_1}{X_2}}}$$

The following relationship is found using an iterative method.

$$X_1 = 0.026X_2 \tag{12}$$

Solving equation (9) $(Y = y' = 8{,}000)$ and (12) for X_1 and X_2 gives

$$X_1^o = 325 \text{ manhours}$$
$$X_2^o = 12{,}500 \text{ dollars}$$

The optimal investment is thus 12,500 dollars. However, refering to Table 2, we would select a single-spindle automatic lathe with a total cost of $11,500.

Method 2. Equation (2) was obtained in previous chapter

$$t = \frac{p}{I^q} \tag{2}$$

in which I = dollars invested in machine-tool

t = unit time in minutes per piece.

On the other hand Y= the demand or output and X_1 the number of manhours to produce the demand Y. X_1 can also be written as

$$X_1 = \frac{Y.t}{60} \tag{13}$$

(13) combined with (2) gives:

$$X_1 = \frac{Y}{60}\ \frac{p}{I^q}$$

let $I = X_2$, and hence

$$Y = \frac{60}{p} X_1 X_2^{q} \tag{14}$$

or the second equation for our production function. In the example the values for p and q were respectively

$$p = 1,723 \text{ X}10^4$$

$$q = 1.67$$

thus equation (14) becomes

$$Y = \frac{60}{1,723\text{X}10^4} X_1 X_2^{1.67}$$

or

$$Y = 3.48 \text{ X } 10^{-6} X_1 X_2^{1.67} \tag{15}$$

Again minimize the cost equation (7) subject to the constraint (15) $Y = y'$ (a constant). The Lagrangian multiplier technique will give the answer.

$$f_1 = \frac{\partial f}{\partial x_1} = 3.48 \text{ X } 10^{-6} X_2^{1.67}$$

$$f_2 = \frac{\partial f}{\partial x_2} = (1.67)(3.48) \text{ X } 10^{-6} X_1 X_2^{0.67}$$

Substituting these values together with the values of C_1 and C_2 in equation (11) gives:

$$\frac{3.75}{3.48\text{X}10^{-6}X_2^{1.67}} = \frac{0.22526}{(1.67)(3.48)\text{X}^{-6}X_1X_2^{0.67}}$$

or

$$\frac{3.75}{X_2} = \frac{0.22526}{1.67X_1}$$

or

$$X_1 = 0.036X_2 \tag{16}$$

Solving equation (15) (y'=8,000) and (16) for X_1 and X_2 gives

$$X_1 = 421 \text{ manhours}$$

$$X_2 = 11,713 \text{ dollars}$$

The optimal investment using method 2 is 11,713 dollars. This result will again lead to the selection of a single spindle automatic lathe as shown in Table 2.

Comparison of the Two Methods

The first method gives a more accurate representation of the curve, but the calculations that are involved to find the parameters and the optimal solution are more complex. Using the second method both the equation (15) and the optimal solution are easier to obtain.

The result obtained with the first method is more accurate as can be shown graphically.

Graphical Solution

Figure 10 shows the production curve for y'=8,000 pieces. On the same figure draw the iso-cost curves represented by equation (6) ($C = C_1X_1+C_2X_2$). Drawing a line parallel to the cost function and tangent to the production curve determines the minimum point. This has been done on Figure 11 and the minimum point (X_1', X_2') is X_1^1 = 325 manhours and X_2^1 = 12,500 dollars.

CHAPTER IV

INVESTMENT RESTRAINT AND SENSITIVITY OF THE MODEL

Restraint on Investment

Previously two approaches were developed to determine the theoretical optimal investment ($11,654 in the example, based on model I). In practice however, it happens that the company cannot afford to invest that optimal amount, because it does not have the money or prefers to invest in other projects at greater rates of return.

Assume that the most the company can invest is $8,000. As shown in Table 2, the price of a turret lathe is close to $8,000, so that a turret lathe is purchased. According to this, the manufacturing cost is $0.624 a piece (value from Table 2). The total cost to manufacture the whole order of 8,000 pieces is then: 0.624 x 8,000 = 4,992 dollars. The total manufacturing cost, using the optimal method was: 0.4738 x 8,000 = 3,790.40 dollars. Therefore, choosing the turret lathe in preference to a single spindle automatic screw machine increases the manufacturing cost by $1,201.60 or ($4,992 - $3,790.60).

If the company wants to invest the difference, 3,765.25 dollars, or (11,500 - 7,734.75; values from Table 2) in another project, then that project should have a rate of return of at least 31.91 per cent.

$$\frac{1,201.60}{3,765.25} \times 100 = 31.91 \text{ per cent}$$

If this return or better is not being obtained from the capital denied to machine-investment, then management's policy of restraining machine investment to invest elsewhere is improper. But if the optimal capital required is not fully available this situation is unavoidable.

Sensivity

Determination of the optimal investment in the preceding chapters was based on the assumption that the factors Q = demand for the product, C_1 = cost of labor and overhead and C_2 = capital recovery cost were known. However, what happens when one or all of these factors change in value?

1. The demand Q

In the example the forecast for the item shown on Figure 4 was expected to be 8,000 pieces a year. However, later it turns out that only a sale of 7,000 items a year may be expected. Or the demand was overestimated by 1,000 a year. What effect will this have on the solution to the problem? Refer to the graphical approach of the production theory. In Chapter 3 it was shown that an iso-product chart could be drawn with each curve corresponding to a different level of output. Thus on the new chart, draw the production curve for an output of 7,000 units. With the help of Table 2 set up Table 4 and draw a curve through the four determinable points as shown on Figure 12.

Table 4. Manufacturing Time

Time per piece in minutes	Time for 7,000 pieces in hours	Time for 8,000 pieces in hours
15	1,750	2,000
6.5	758	866
3	350	400
1.2	140	160

A line parallel to the cost curve and tangent to the new production curve gives the optimal investment ($11,500). There is no change

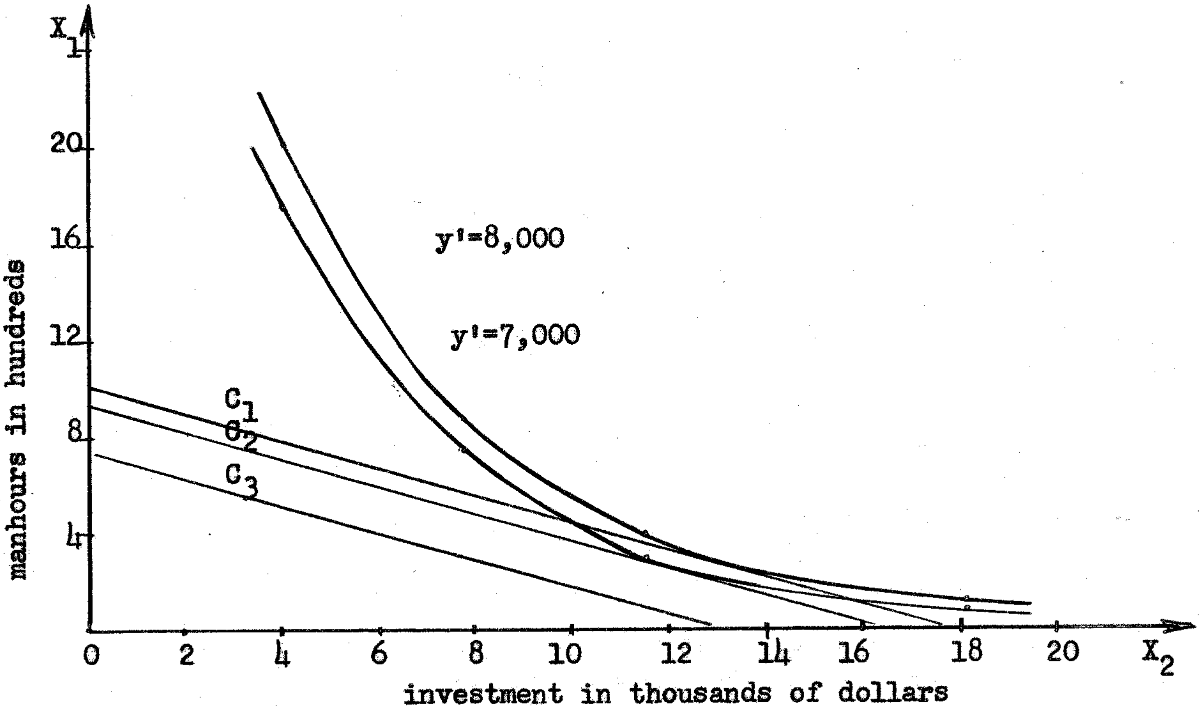

Figure 12. Optimal Point for a Demand of 7,000 Units

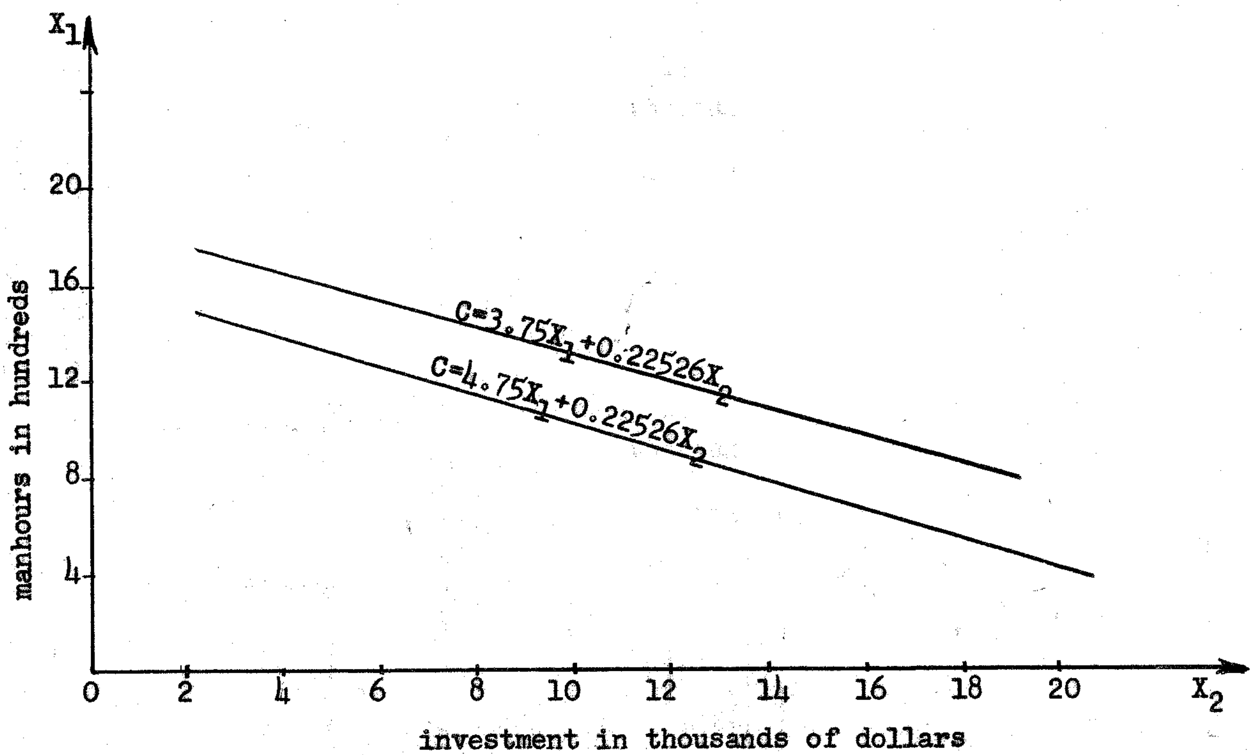

Figure 13. Change of One of the Cost Factors

in the final decision, a single-spindle automatic screw machine (cost \$11,500) will be purchased. This will not always happen and if the change in the variable (here demand Q) makes the decision uncertain, other factors should be taken into account such as the kind of production planned or the possibility of using the machine for other products, etc.

2. The Labor Cost C_1 and the Capital Recovery Cost C_2

The factors C_1 and C_2 appear only in the equation of the cost function. ($C = C_1X_1+C_2X_2$). The cost function is a linear equation and a change of one of the factors C_1 or C_2 changes only the slope of the line. For example: let the cost of labor and overhead be \$4.75 instead of \$3.75 per hour as previously used. The cost function then becomes $C = 4.75X_1+ 0.22526X_2$, as shown in Figure 13. Thus it only changes the slope of the line. The point of tangency of the cost function with the production curve gives the new optimal investment. It should be noted that the flexibility has only been shown utilizing the production theory approach but the same method could be applied using the economic balance approach.

CHAPTER V

LIMITATIONS IN THE APPLICATION OF THE MODEL

Before we can apply the model in a particular situation, there is an important factor that must be considered, namely the kind of production the manufacturing firm is engaged in.

Generally speaking there are two broad classifications of types of production: (1) production to order, and (2) production to stock. Further in each of these two types of production three subclassifications usually appear: (1) jobbing production, (2) continuous production, and (3) intermittent production. If production is not started except after customer's order is received and the quantity produced is strictly the order quantity (no stocks), then production is said to be to order. On the other hand, if product specifications can be established with reasonable certainty that they will meet different customer needs and desires without dealing directly with individual customers, then production may be undertaken to stock.

The subclassification is related to the expected sales volume or quantity of the product demanded by customers per period of time, say, one year. If the demand for an output of a particular item is expected to be very low, production will be done economically on a jobbing basis without pre-process engineering. If the demand is expected to be very large, production will be most economically done on a continuous basis, with pre-planning and engineering. If the expected demand is not of such volume as to economically permit either continuous production or jobbing production but something in between, intermittent production is the economically appropriate type (usually production in economic batches or run sizes).

The expected volume of output dictates the nature of the inputs used in the production function. Obviously, it would be uneconomical to use a high-priced machine tool to perform an operation on only a few items a year, with the machine tool standing idle much of the time.

The next step is to determine the nature of the inputs to be used to the conversion processes in each type of production.

Production of Goods to Order

1. Jobbing Production of Goods to Order

The relatively wide variation of product specifications demanded in the product mix makes it uncertain as to the types of conversion operations or manufacturing processes to employ in production. Therefore, it is better to strive for the highest degree of flexibility in process equipment. Hence, general purpose machine-tools are purchased, or rented. Subcontracting should be considered before implementing a decision causing excessive investment in machine-tools.

2. Continuous Production of Goods to Order

The volume of the order is of such size that the operations can continue over a long period of time between setup changes (e.g., when product line setup is not used). Because the volume of the demand is so large, physical facilities can be specialized in their use. "Specialized in use" meaning that general purpose machines can be devoted completely to the production of a particular component part. The volume is so large that the machine's idle time can be minimized (with good maintenance policy). Sometimes the high volume indicates the use of some special-purpose machines along with the general-purpose machines, especially when some of the operations on the different parts are repetitive. In any case the developed model can still be used to determine how much money should be invested in machine-tools.

3. Intermittent Production of Goods to Order

As the volume of output to a given set of specifications is not very large, physical facilities usually cannot be special-purpose nor specialized in use as in a production line. General-purpose equipment is used and grouped departmentally by process rather than by product as is done in continuous production. In this respect it is like jobbing production, except that economic batch-run quantities may be more economical than running the entire quantity of the order at a single time period.

Production of Goods for Stock

1. Jobbing Production of Goods for Stock

This type of production is relatively rare. If product characteristics are known in advance, even though the market demand is low, the producer can compete more effectively by producing the product intermittently to fill orders on hand only or replace stock as it is depleted by the small order withdrawals from stock. Jobbing for stock only differs from jobbing to order in that the production can be planned.

2. Continuous Production of Goods for Stock

Extremely high volume makes continuous production the most economical. Outputs are standardized, so that flexibility of inputs is not required. Special-purpose equipment is used, and devoted entirely to one particular output. In fact physical facilities are so highly specialized that they are often fully automated equipment. The degree of automation will of course depend on the volume of production. Here again the previously developed models can be used advantageously. For example, if the demand for a product is high, a continuous production line using general-purpose equipment, or a continuous production line using partly general-purpose and partly special-purpose machines, or an automated production line (transfer-machine) operated by electronic equipment could be used to manufacture the part. The composition of the line and the number of machines in the line can be varied by interchanging general-purpose machines and special-purpose machines. The exact composition of the product line that is machine types can be determined by use of the economic balance approach or production theory approach. By use of one of these approaches the optimal amount to invest can be determined which in turn dictates the composition of the line. For an example, see appendix.

3. Intermittent Production of Goods for Stock

Product specifications are known in advance and are standardized. Hence, goods are produced for stock for competitive reasons. The volume, however, is not sufficiently large to economically employ continuous operations. Thus the goods are produced in lots or economic run sizes, to replace stock as it is depleted through shipments to customers.

Technologically appropriate physical facilities cannot be kept busy continuously on one output or product as in continuous production. The degree of similarity that exists between the different products to be made will also determine the lay-out. If some of the successive operations are repetitive but on different components, a more specialized machine can be purchased. For instance, a component part requires successive operations on a milling machine, a drill, a lathe, and a threader, while another component part requires a milling machine, a lathe and boring machine. The lathe operation appears as a common operation for both parts so that a turret lathe may perhaps be considered instead of an engine lathe. In other words, explode the product mix into machine loads for the various machines. Common parts and processes will cause high volume on a particular process. The economic model can also determine in this case the amount that should be invested.

The fact that the component may be produced in economic lot sizes across the different machines dictates lay-out by process rather than by product.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATION

The objective of this study was to develop a model to determine the machine-tool investment that minimizes the unit cost for a certain demand. Two models were developed, incorporating the relationships between cost, demand and investment, e.g., the economic balance model and the production theory model. The first model expressed the relationship that exists between the unit cost, the demand and the investment, and the second model the relationship between the total cost, the demand and the investment. For both models, graphical and analytical approaches were made.

It was concluded that the graphical approach gave a more accurate solution to the problem, as the analytical approach gave only an approximation of the production curve. In the economic balance approach an approximation was obtained by fitting a line, on logarithmic paper, through a set of scatter points, while the graphical approach gave a more realistic representation of the curve.

Using the production theory, two analytical methods were set up. The first method, through equation 9, gave a more accurate representation of the production curve but the calculation involved to determine the parameters of the production function and the optimal investment were more complex. Using the second method both equation 15 and the optimal solution were found to be easier to obtain. However, the second method is dependent on the first model, especially for the equation of the production function, so that the same approximation of fitting a line through scatter points limits the accuracy of this method also.

The graphical approach to the production theory can be easily achieved because it only requires drawing a line parallel to the cost function and tangent to the production curve to determine the optimal investment. The graphical approach to the economic balance theory requires the calculation of the values in Table 2, the capital recovery cost per piece and the labor and overhead cost per piece. However, none of these calculations are necessary when using the production theory approach.

Finally, the graphical solution of both methods yields the same accuracy but the production theory model is the easiest to conceive.

It should be noted however that other factors, for instance, the physical characteristics of the machine-tools and the types of production the firm may be engaged in, should be considered before making a final decision.

It should also be noted that the fitting of the curve to the empirical data used in the economic balance approach in the example was based on only four points. However, in practice many more machines should be considered so that a closer approximation will result.

It is recommended that experimentation be done in the future to find a general expression for an equation that may express the relationships between process time and the machine investment.

APPENDIX

Example of Continuous Production

The United States Carbine, Caliber 0.30, M1 consists of numerous component parts, one of which is "the ejector", as shown on Figure 14.

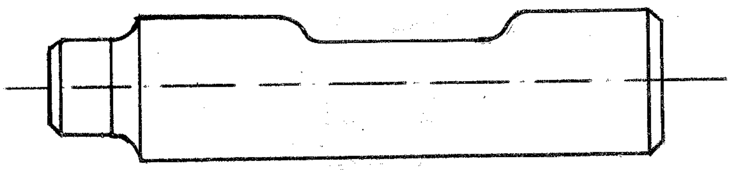

Figure 14. The Ejector

The data were obtained from a "Description of Manufacture" prepared by the Inland Manufacturing Division of the General Motors Corporation in Dayton, Ohio.

The demand is assumed to be 650 Carbines daily, over a period of five years, plus spare parts. There is one ejector per Carbine, and the contract requires eight spares per 100 carbines. Previous processing has shown that on the average eleven percent are rejected by inspection. The number of parts to be produced daily is:

$$\frac{650(1+0.08)}{1-0.11} = 789 \text{ parts}$$

The question is, how to process the part and what machine-tools should be selected (either now available or to be purchased).

The first step is to prepare the production routing. Three different routings will be compared for three different feasible and possible processing sequences.

Route Sheet 1

	Operations	Manhours per 100 pieces	Machiney and Equipment	Unit Cost $
1	Form and cut off	1.12	Turret Lathe	6,500.00
3	Mill slot	1.00	Milling Machine	9,300.00
4	Wash	0.02	Detrex Washer	600.00
5	Tumble	0.06	Tumbling Barrel	700.00
6	Heat treat	0.03	Electric Box Furnace	2,500.00
	Total	2.43	Total	$19,600.00

Operation one takes over eight hours to produce the 789 pieces, so that two turret lathes will be needed, or one turret lathe and an engine lathe. If the last approach is used, the bulk is processed on the turret lathe and the balance on the engine lathe. (Or, of course, one turret lathe and overtime could be used).

In one shift of eight hours duration the turret lathe can produce 700 pieces, the remaining 89 will be manufactured on the engine lathe. The engine lathe requires 4.58 hours to produce 100 pieces and the cost of an engine lathe is $4,000.

The labor requirements are then, one man to operate the turret lathe, one man for the milling machine, and one man to perform all other operations. In total three men and an investment of $23,600 (19,600 + 4,000) are required.

Route Sheet 2

	Operations	Manhours per 100 pieces	Machinery and Equipment	Unit Cost $
1	Form and cut off	0.52	Automatic Screw Machine	$13,450.00
2	Mill and cut off teat	.25	Stub Press	2,285.00
3	Mill slot	1.00	Milling Machine	9,300.00
4	Wash	0.02	Dextrex Washer	600.00
5	Tumble	0.06	Tumbling Barrel	700.00
6	Heat treat	0.03	Electric Box Furnace	2,500.00
	Total	1.88	Total	$28,835.00

Operation two is supplementary because the automatic screw machine leaves a little teat after the cutting process. This can be avoided with a turret lathe.

Theoretically two men will be sufficient to perform the work of Routing 2, although in practice three men will be working on these operations, but will be performing work additional to these operations. Thus two men and an investment of $28,835 are estimated to be required.

Route Sheet 3

Five total operations can be performed on one "transfer machine", specially designed for this part. In this case one man can easily handle the job but the machine-tool investment is estimated to be $60,000.

The second step is to set up the cost equation. The cost equation is given by $C = C_1X_1 + C_2X_2$, as previously shown in Chapter III.

C_1 = labor and overhead cost per hour ($3.75)

C_2 = capital recovery factor = 0.33438 (for five years at a rate

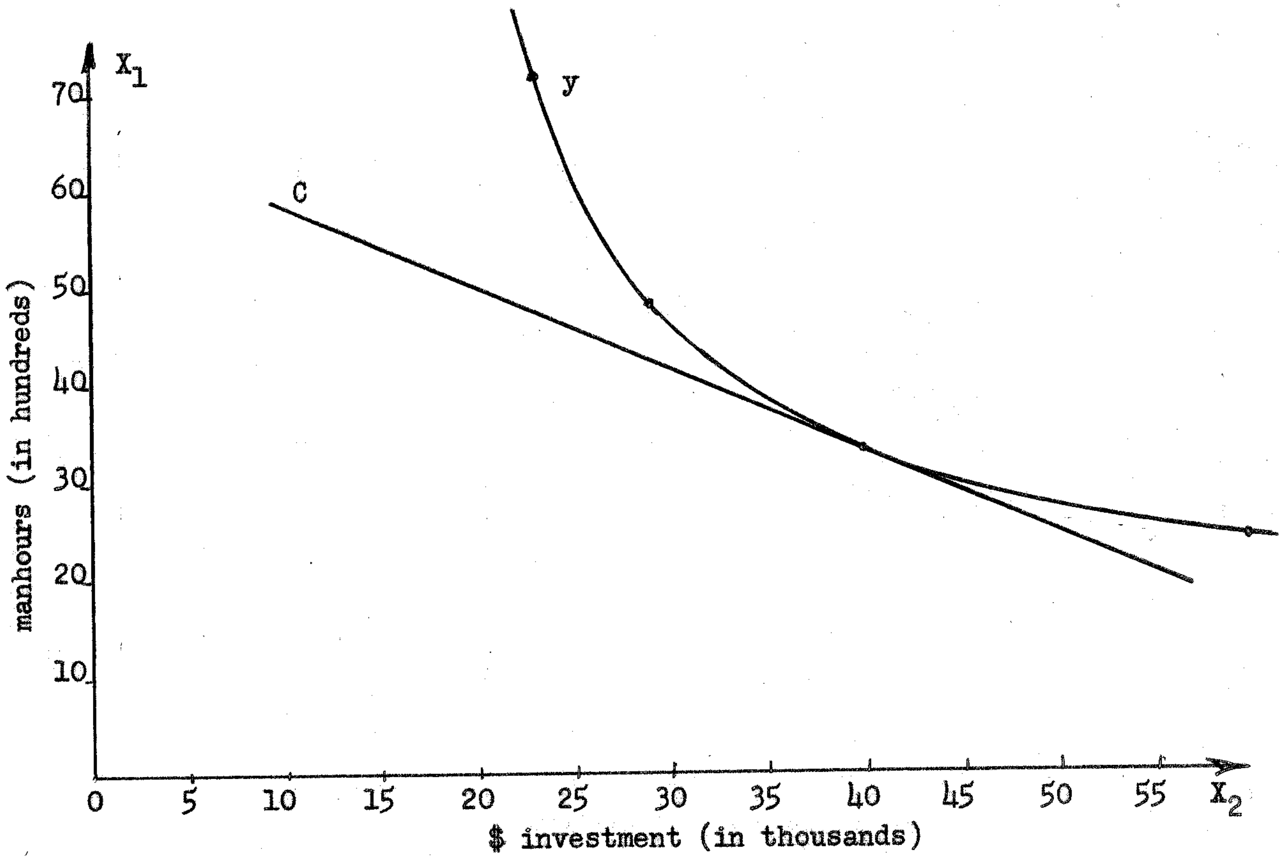

Figure 15. Graphical Solution for the Ejector

of return of 20 per cent on invested capital)

$$C = 3.75X_1 + 0.22526X_2$$

The graphical approach will be used to solve the problem. The solution is shown in Figure 15. It follows then that $40,000 only can be invested in machine-tools to produce the ejector. Before making a decision as to process or machine-tool selection, additional information should be sought as to possible extension of the contract or the possibility of an increased daily production of the current contract. This added information as to future production requirements may justify the procurement of the transfer-machine. However, if chances of renewal of the current orders are slim, the second production plan, Route Sheet 2, should be favored.

Remark:

In solving the above example the graphical solution of the production theory approach has been used. It would be possible however to use either the economic balance approach or the analytical approach of the production theory.

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